

Mathematical Modeling: An Exciting and Challenging Change!

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May 19, 2012

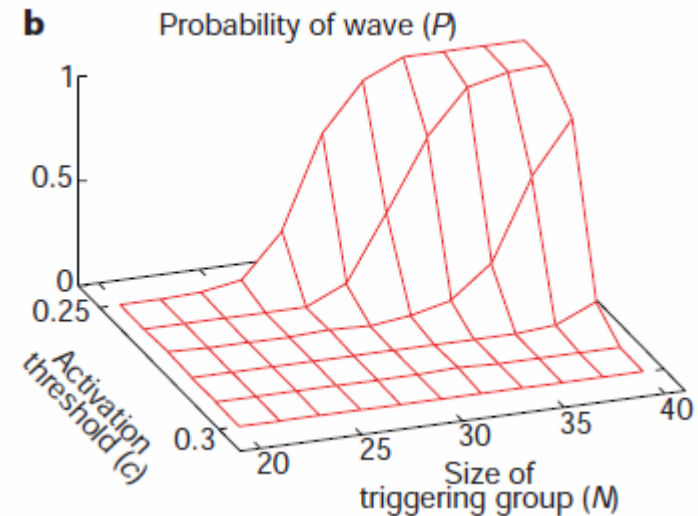
The Wave

- If our entire school was sitting in the auditorium, how long would it take us to do a Wave?
- The front row in our auditorium seats 30 students. Every row after that seats 4 more students than the row before it. There are 20 rows of seats altogether. If every seat is full, and students average 1.5 seconds to do the Wave, how much time will it take for the entire auditorium to complete the Wave?
- Our auditorium seats 325 students. If every seat is full, and students average 1.5 seconds to do the Wave, how much time will it take for the entire auditorium to complete the Wave?

- Mathematical Modeling
 - Identifying important quantities in a practical situation
 - Making assumptions and approximations to simplify a complicated situation
 - Mapping relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
 - Analyzing relationships mathematically to draw conclusions
 - Interpreting mathematical results in the context of the situation
 - Reflecting on whether the results make sense
 - Improving the model if it has not served its purpose.
- Open-ended problem solving
 - Goal: to identify and implement appropriate strategies, using metacognition and problem solving heuristics
 - Students are expected to persevere and look for strategies to solve problems when they don't know a procedure
- Word problems
 - Goal: to identify appropriate procedures and solve context-based problems
 - Students are expected to know a procedure to solve the problem

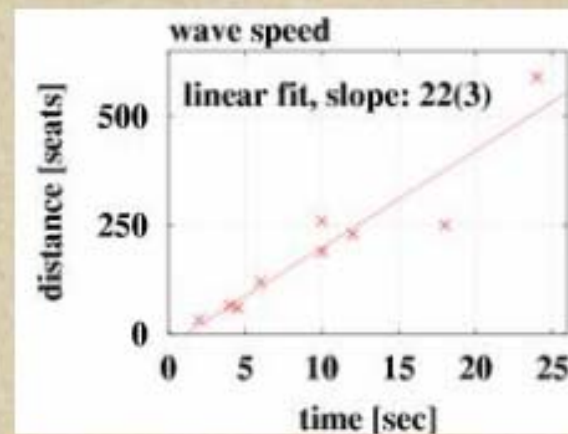
The Mexican Wave

<http://www.youtube.com/watch?v=LwpgUBZ-FIA>

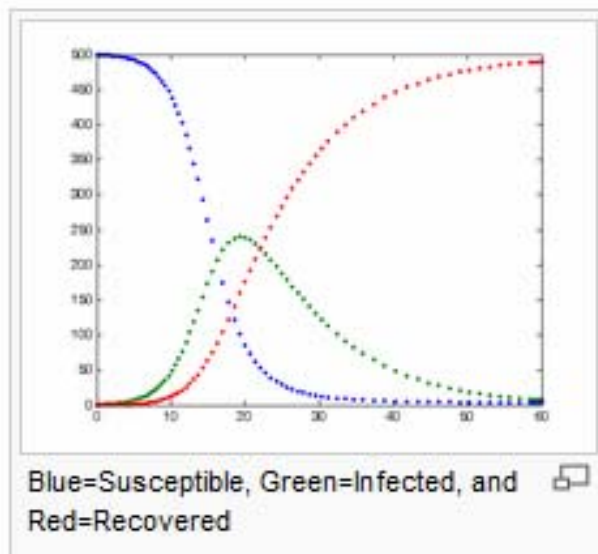


Speed of the wave

The distance vs. time relationship for 9 different waves (from video recordings, data by T. Seidel). The slope of the fitted line gives the average speed of the wave: (22 ± 3) seats/sec.



Modeling the progress of an epidemic in a large population

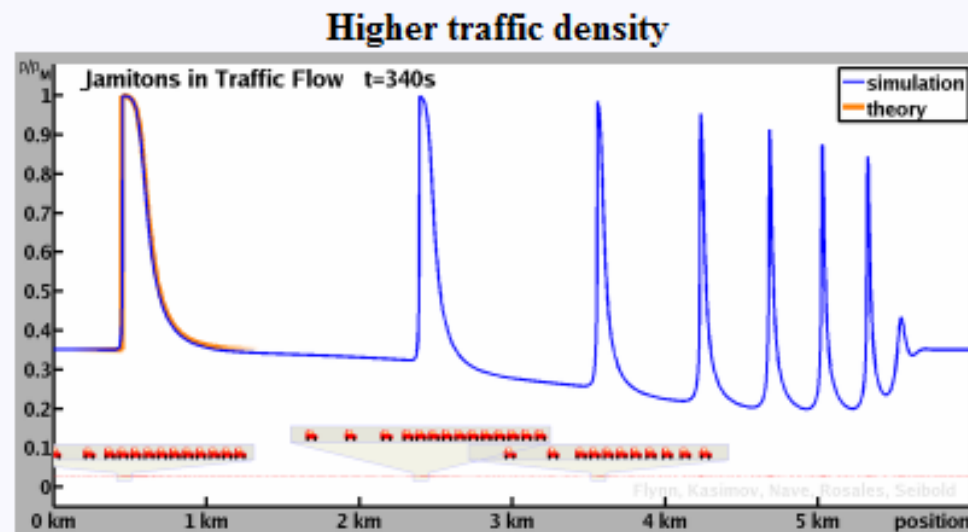


$$\begin{aligned}\frac{dS}{dt} &= -\beta IS \\ \frac{dI}{dt} &= \beta IS - \nu I \\ \frac{dR}{dt} &= \nu I\end{aligned}$$

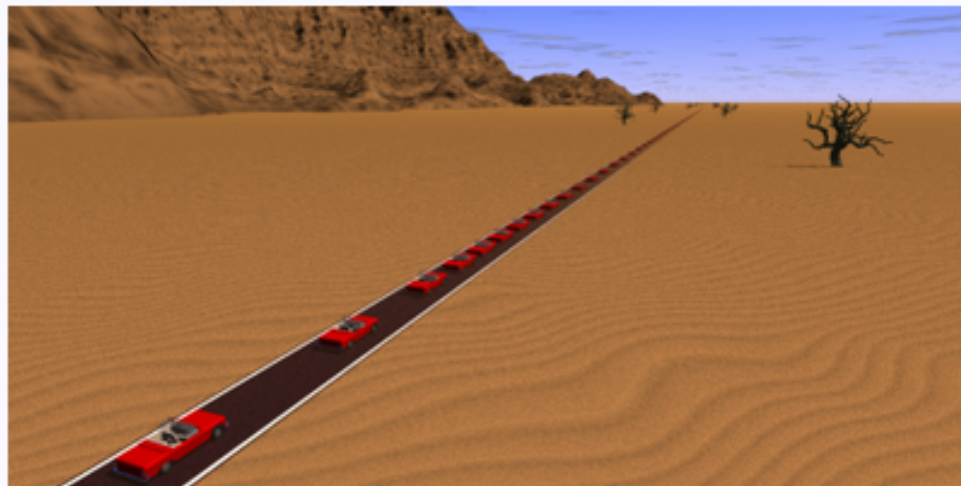
- Simple compartmental model for diseases such as measles, mumps, rubella

http://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology

Traffic Jamitons



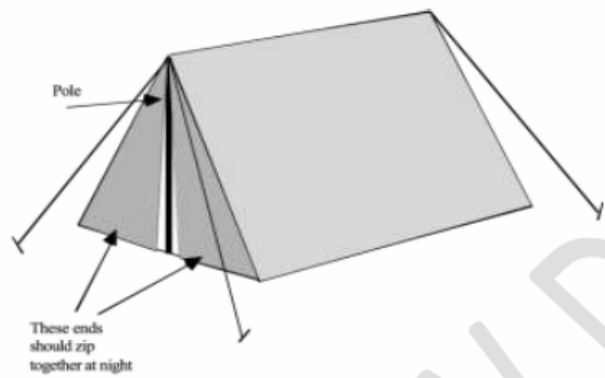
[Download Video](#) (divx, 9MB)



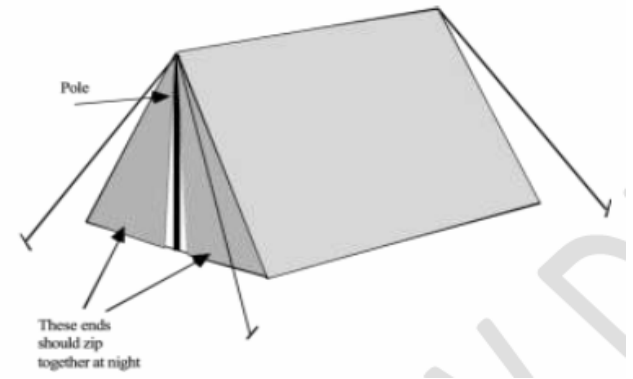
[Download Video](#) (divx, 18MB)

Design a Tent

- How many square feet of plastic would you need to make a tent if the floor is 4' x 7' and the height is 5'?



How will we help prepare students to make the shift to Common Core standards?



- CST: How many square feet of plastic would you need to make a tent if the floor is 4' x 7' and the height is 5'?
- Common Core: Your task is to design a 2-person tent. Your design must satisfy these conditions:
 - It must be big enough for someone to move around in while kneeling, and big enough for all the stuff for 2 people.
 - Make drawings to show how you will cut the plastic for the ground cover, and how you will cut the material for the sides of the tent (in one continuous piece).
 - Make sure you show the measures of all lengths and angles, and explain why you have made the choices you have made.

Design a Tent

Your task is to design a 2-person tent like the one in the picture.

Your design must satisfy these conditions:

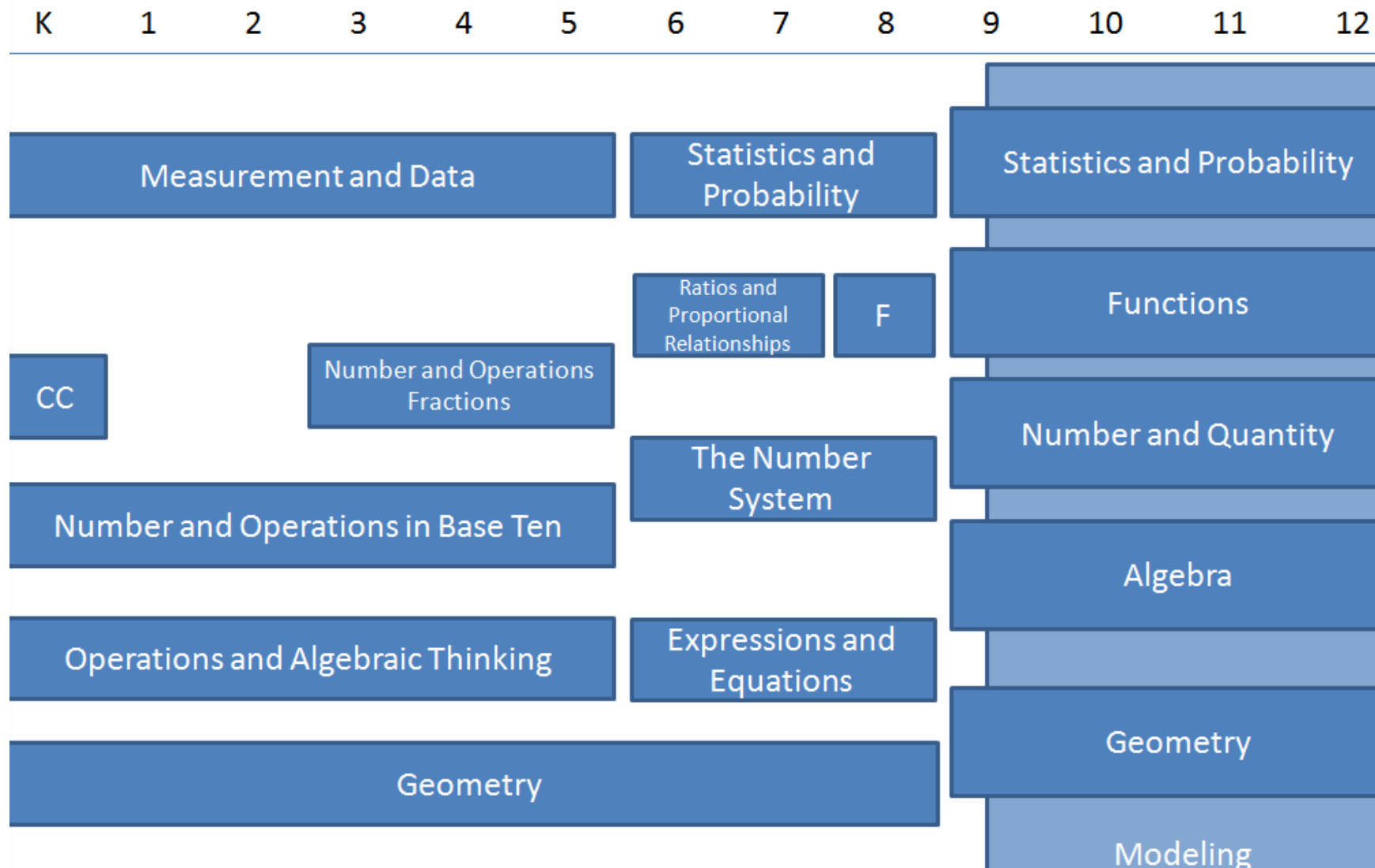
- It must be big enough for someone to move around in while kneeling down, and big enough for all their stuff.
- The bottom of the tent will be made from a thick rectangle of plastic.
- The sloping sides and the two ends will be made from a single, large sheet of material.
- Two vertical tent poles will hold the whole tent up.

Make drawings to show how you will cut the plastic and the material.

Make sure you show the measures of all relevant lengths and angles clearly on your drawings, and explain why you have made the choices you have made.



Common Core Math Domains

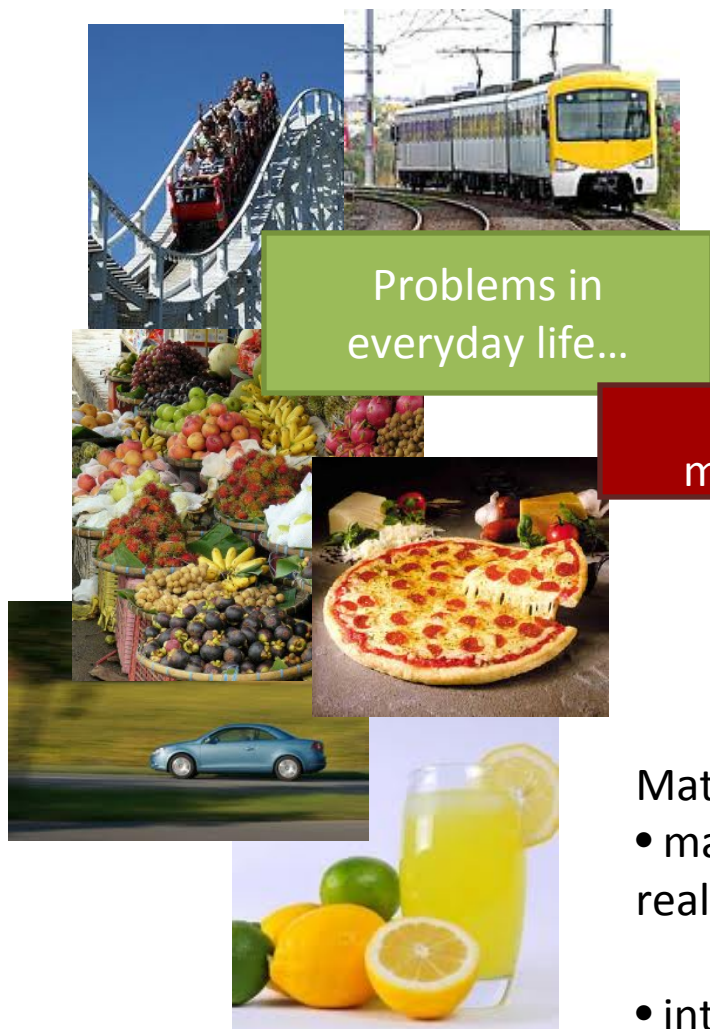


From CCSSM Clickable Map found at <http://commoncoretools.me/tools/>

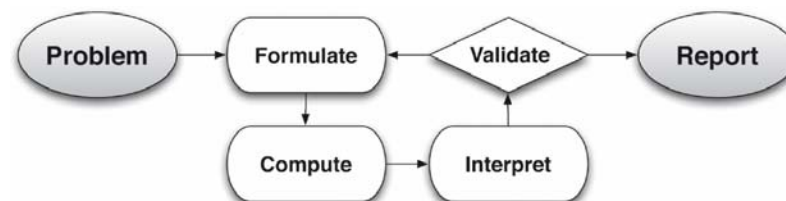
Common Core Math Practices

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

SMP 4: Model with mathematics



...reasoned using mathematical methods



Mathematically proficient students

- make assumptions and approximations to simplify a situation, realizing these may need revision later
- interpret mathematical results in the context of the situation and reflect on whether they make sense

The Snack Chip Consumer Guide Problem (English, 2002a)

Students are presented with an introductory article on consumer guides, with questions to answer about the article. They are then given the following problem, with various packets of snack chips provided for them.

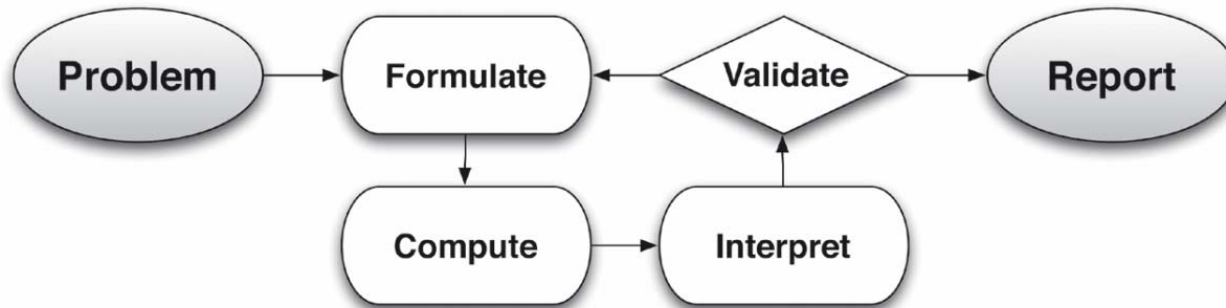
In this investigation, you will be developing a consumer guide to help people determine which type of snack chip is the best to buy. It is your decision what to focus on in your consumer guide. Your consumer guide must help people in choosing any snack chip, not just the ones you use in this activity.

As a whole class, brainstorm some factors or criteria that you might consider when you are trying to work out which chip is the best to buy. Think about what we could mean by *best*. Next, in your groups, discuss the following.

1. Describe the nature or type of factors that the whole class brainstormed. What type of information does each factor give you?
2. How might you categorize the factors?
3. How might you rate the factors to help the consumer determine which packet of snack chips they should buy?

From Mathematical Modelling with Young Learners by Lyn English
at <http://eprints.qut.edu.au/1640/1/Englishchapter.pdf>

A model of mathematical modeling



- Identifying important quantities in a practical situation
- Making assumptions and approximations to simplify a complicated situation
- Mapping relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Analyzing relationships mathematically to draw conclusions
- Interpreting mathematical results in the context of the situation
- Reflecting on whether the results make sense
- Improving the model if it has not served its purpose.

Students need to learn to identify quantities

A **quantity** is anything (an object, event, or quality thereof) that can be measured or counted. The **value** of a quantity is its measure or the number of items that are counted. A value of a quantity involves a number and a unit of measure or number of units.

- The length of this room
- The number of people in this room
- The temperature in the room
- The noise level
- The difference between the number of people in this room and the number of people in Room 2009

What Car to Buy?

Carl and his mother have been out shopping for cars. Carl wants a car that will be fun to drive around in, gets good gas mileage, but doesn't cost too much. But Carl's mother, who is going to help pay for the car, wants him to have a car that is reliable and safe. Your job is to create a list for Carl and a list for his mother showing which cars are the best. Then they will have to decide which one to buy!

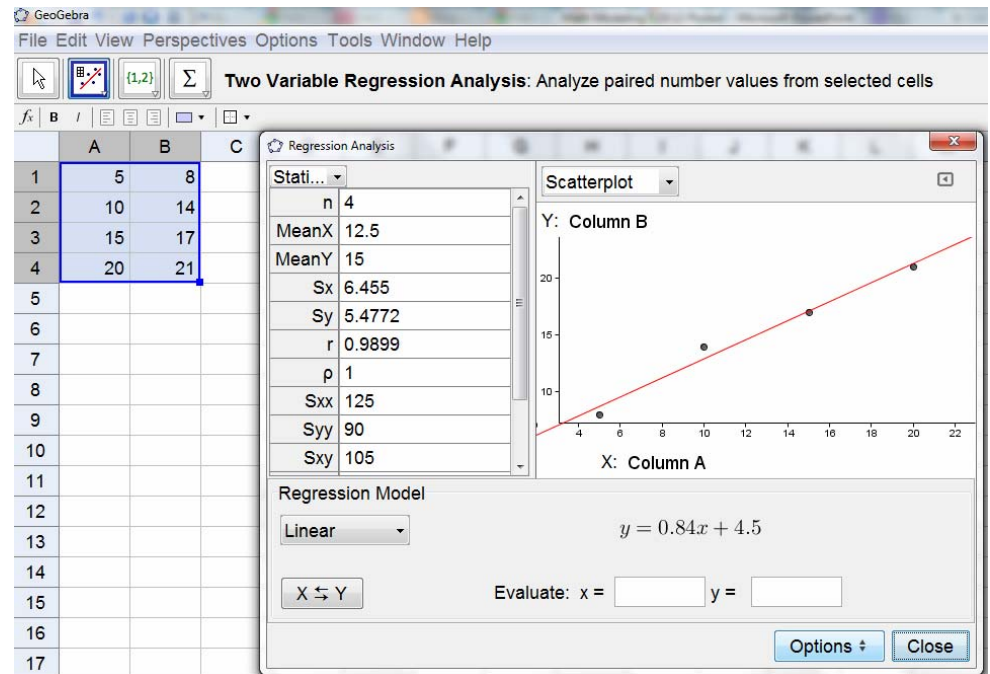
Car Information

Car	Year	Cost	Color	Mileage	litres/ 100 km City	Features	Body Style
Nissan Silva	1992	10,000	Navy Blue	96,000	10	Rear Spoiler, Power Windows, Power Steering, CD Player, Alloy Wheels, Alarm	Coup
Ford Capri Turbo	1989	8,200	Red	105,000	9	Rear Spoiler, Power Windows, Power Steering, CD Player, Alloy Wheels, Alarm Rear Spoiler, Power	Convertible

From Mathematical Modelling in the Primary School by Lyn English
at <http://www.merga.net.au/documents/RP232004.pdf>

Students need to learn to use technology strategically

- Spreadsheets
- Dynamic geometry and algebra inquiry software (www.geogebra.org)
- Calculators

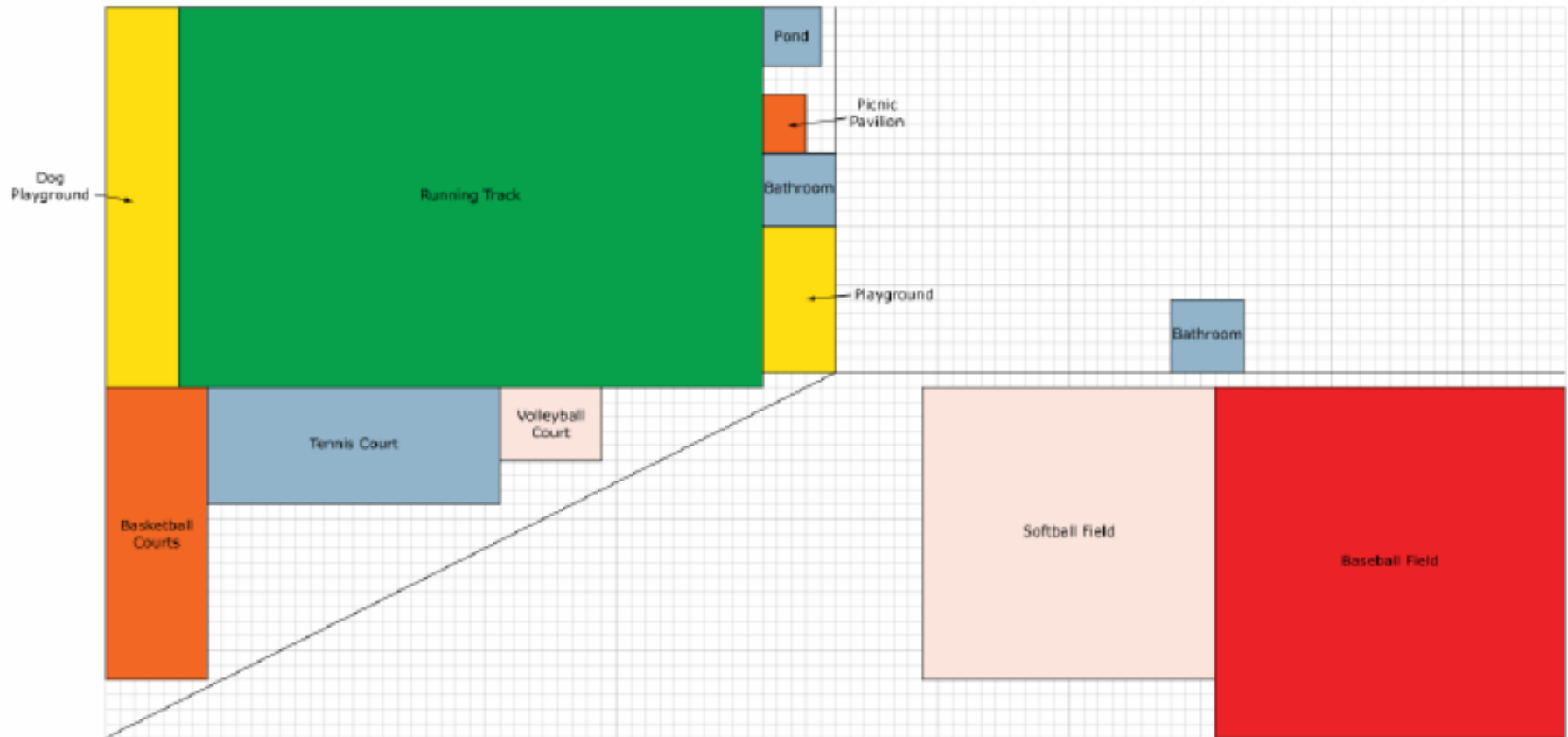


Spreadsheet

	A	B	C	D	E	F	G
1							
2			Estimated Cost	Length (yards)	Width (yards)	Area	
3		Open Areas				56,250	
4		Paths, Benches, and Bathrooms				3750	
5		4 Basketball Courts	\$200,000	100	35	3500	
6		2 Volleyball Courts	\$75,000	35	25	875	
7							
8		Running Track	\$380,000	200	130	26000	
9							
10		4 Tennis Courts	\$200,000	100	40	4000	
11		Baseball Field	\$100,000	120	120	14400	
12		Softball Field	\$75,000	100	100	10000	
13		Playground	\$70,000	50	25	1250	
14		Dog Playground	\$30,000	100	25	2500	
15		Picnic Pavilion	\$70,000	20	15	300	
16		Pond	\$30,000	20	20	400	
17							
18		Total used	\$1,230,000			123225	
19							
20		Amount available	\$1,300,000			125000	

Smarter Balanced Assessment Consortium Mathematics Claim #4 Rationale and Assessment Items –
 Reposted at <https://webfiles.uci.edu/vhenry/Math%20Modeling%20Specifications%20and%20Sample%20Items.pdf?uniq=n6887f>

Model



Smarter Balanced Assessment Consortium Mathematics Claim #4 Rationale and Assessment Items –
Reposted at <https://webfiles.uci.edu/vhenry/Math%20Modeling%20Specifications%20and%20Sample%20Items.pdf?uniq=n6887f>

Students need to learn to communicate mathematically

Sample Top-Score Response:

Question 1

First I need to determine the total length of the path and the number of benches needed.

For the length of the path, one section is $250 \div 2 = 125$ yards long, and another section is $500 \div 2 = 250$ yards long. I need to use the Pythagorean theorem to find the length of the third section.

$$a^2 + b^2 = c^2$$

$$125^2 + 250^2 = c^2$$

$$15625 + 62500 = c^2$$

$$78125 = c^2$$

$$c \approx 280$$

The length of the third section is about 280 yards, so the total length of the path is 655 yards.

The benches are located about every 25 yards, so the first section needs 5 benches, the second section needs 10, and the third section needs 11, since $280 \div 25 \approx 11.2$. There are also 3 benches in the middle at the intersection of the paths for a total of 29 benches.

Smarter Balanced Assessment Consortium Mathematics Claim #4 Rationale and Assessment Items –

Reposted at <https://webfiles.uci.edu/vhenry/Math%20Modeling%20Specifications%20and%20Sample%20Items.pdf?uniq=n6887f>

I will multiply the cost per yard of the path by the number of yards in the path, I will add the cost per light and the cost per bench and multiply the sum by 29 because there are 29 of each, and I will multiply the cost per restroom by the number of restrooms, and then add that all up together. Or, in the case of Coleman Contractors, I will just add the numbers given.

So, for Aguilar Construction, the cost is:

$$30 \times 655 = \$19,650$$

$$225 + 275 = 500$$

$$500 \times 29 = \$14,500$$

$$2 \times 120750 = \$241,500$$

$$19650 + 14500 + 241500 = \$275,650$$

For Benedetto & Son, the cost is:

$$20 \times 655 = \$13,100$$

$$335 + 365 = 700$$

$$700 \times 29 = \$20,300$$

$$2 \times 108,300 = \$216,600$$

$$13100 + 20300 + 216600 = \$250,000$$

Smarter Balanced Assessment Consortium Mathematics Claim #4 Rationale and Assessment Items –

Reposted at <https://webfiles.uci.edu/vhenry/Math%20Modeling%20Specifications%20and%20Sample%20Items.pdf?uniq=n6887f>

Conclusion

First I calculated the area of each facility on the spreadsheet. I also included the minimum area that needed to remain open space, and the space taken up by the paths, benches, and restrooms. I found the total cost and the total area, and I saw that my total cost was way more than \$1,300,000, and my area was more than 125,000 square yards also.

Then I decided to make up my mind about the track and turf field, because they took up a lot of area, they very expensive, and they were confusing on the chart since things were mentioned more than once. If I included the track and turf field, I would have only \$20,000 to do anything else, which didn't seem smart. Since cost was over by a lot more than area, I thought cost should be a bigger concern than area. I decided to eliminate the turf field since it cost so much and it seemed like people wanted the track just as much. So I eliminated the turf field and turf field with running track lines from my spreadsheet, leaving me with just the running track. This helped simplify things.

To my surprise, I was already under budget, but I still had a little too much area. I noticed that the community garden took up a lot of area, was pretty cheap, and not many people wanted it, so I deleted that and saw that I was within the limits for money and area. I was not using some of the money, but I figured the foundation could give it to someone else.

Students need to learn to work at all Depth of Knowledge Levels

- **DOK-1 - Recall & Reproduction** - Recall of a fact, term, principle, concept; perform a routine procedure; locate details
- **DOK-2 - Basic Application of Skills/Concepts** - Use of information; conceptual knowledge; select appropriate procedures for a given task; two or more steps with decision points along the way; routine problems; organize/display data; interpret/use simple graphs; summarize; identify main idea; explain relationships; make predictions
- **DOK-3 - Strategic Thinking** - Requires reasoning, or developing a plan or sequence of steps to approach problem; requires decision making or justification; abstract, complex, or non-routine; often more than one possible answer; support solutions or judgments with text evidence
- **DOK-4 - Extended Thinking** - An investigation or application to real world; requires time to research, problem solve, and process multiple conditions of the problem or task; non-routine manipulations; synthesize information across disciplines/content areas/multiple sources

DOK (Depth of Knowledge)

Level 3: Strategic Reasoning

■ **A.** Focus is on reasoning & planning in order to respond (e.g., write an essay, apply in new/novel situation).
B. Complex and abstract thinking is required. **C.** Often need to provide support for reasoning or conclusions drawn. **D.** More than one "correct" response or approach is often possible.

Level 1: Recall

■ **A.** Focus is on specific facts, definitions, details, or using routine procedures (measure, divide, follow recipe, etc.) **B.** Explaining "that..."
C. Can be "difficult" without requiring "deep" content knowledge to respond to item (memorize a complex theory without being able to explain its meaning or apply it to a real work situation)
D. Combination of level ones does NOT = level 2.
E. One right answer

Level 4: Extended Reasoning

■ **A.** Requires complex reasoning, planning, and thinking (generally over extended periods of time) for the investigation. **B.** Assessment activities have multiple steps with extended time provided. **C.** Students may be asked to relate concepts within the content area and among other content areas.
D. Students make real-world applications in new situations.

Level 2: Skill/Concept

■ **A.** Focus is on applying skills and concepts (in a familiar/typical situation), relationships (compare, cause-effect), main ideas. **B.** Requires deeper knowledge than definition
C. Explaining how or why **D.** Making decisions **E.** Estimating, interpreting in order to respond **F.** One right answer



**"He who learns but does not think, is lost!
 He who thinks but does not learn
 is in great danger." Confucius**

Southern Nevada RPDP

6

Table 2: Suggested methods and wait time in relation to the depth-of-knowledge level of the activity.

Level	Suggested method	Typical wait time
DOK-1	Individual student	At least 3 sec, more for ELL or struggling students.
DOK-2	Pair-share/peer-share	Considerably longer than DOK-1.
DOK-3	Formal group activity	On the order of magnitude of ten minutes.
DOK-4	Extended group activity	Multiple classroom sessions

Table 1: The four depth-of-knowledge levels formulated by Norman Webb [4, 5].

Level	Description
DOK-1	Recalls facts, information, or procedures.
DOK-2	Uses information or conceptual knowledge, two or more steps, etc.
DOK-3	Requires reasoning, developing a plan or a sequence of steps, involves some complexity, more than one possible answer.
DOK-4	Requires investigation, time to think, and processing of multiple conditions.

Students need to learn how to use math in the real world

My brother says ...

- **When purchasing a car -- buy used.**

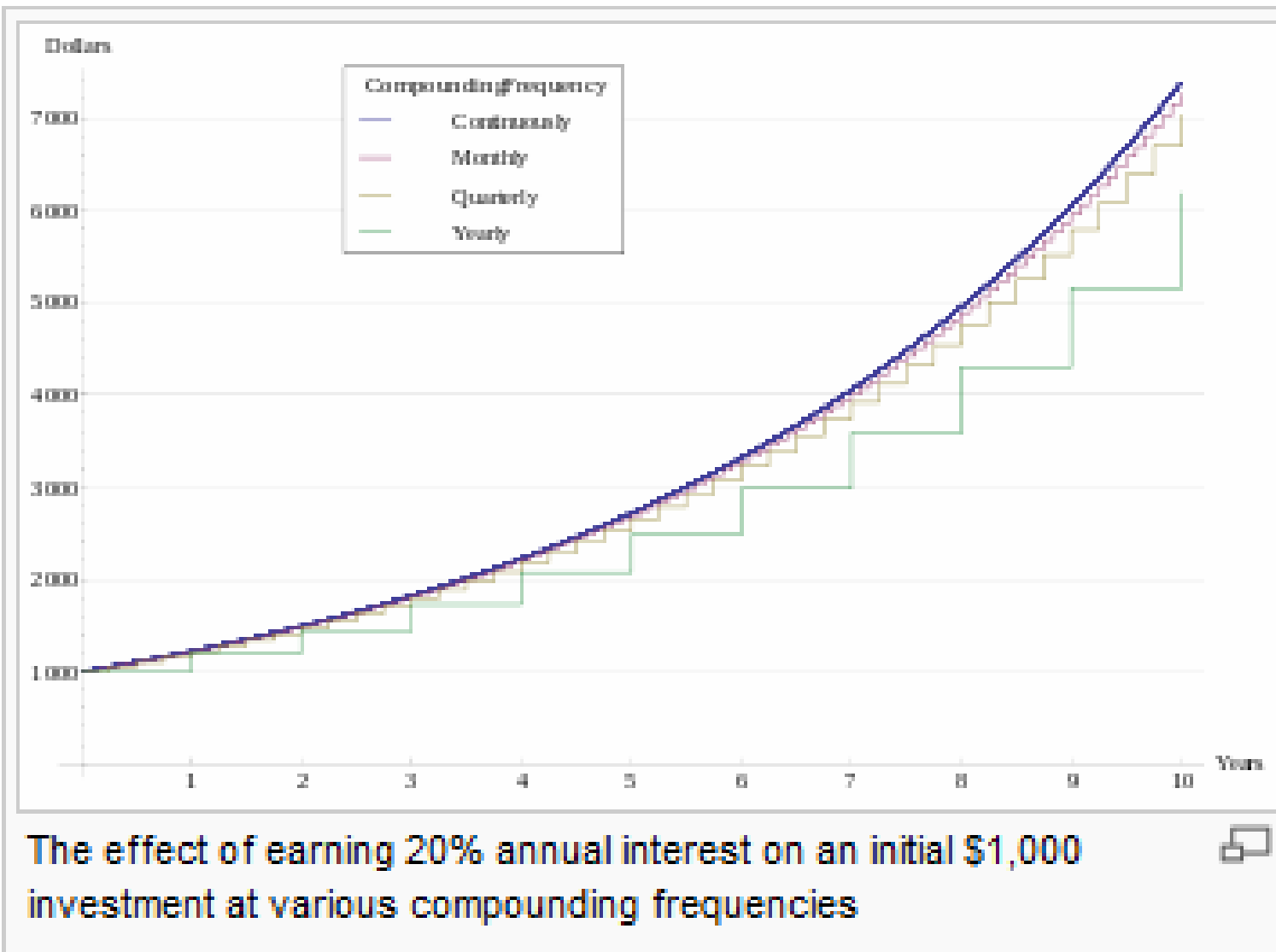
<http://www.getrichslowly.org/blog/2009/03/09/25-favorite-financial-rules-of-thumb/>

Compound Interest

The diagram shows the compound interest formula $A = P \left(1 + \frac{r}{n}\right)^{nt}$ enclosed in a red rectangular box. Various parts of the formula are labeled with arrows pointing to them:

- A : amt invest. over time
- P : principal (initial)
- r : % annual rate (dec)
- n : # of times comp. per year
- t : # of years

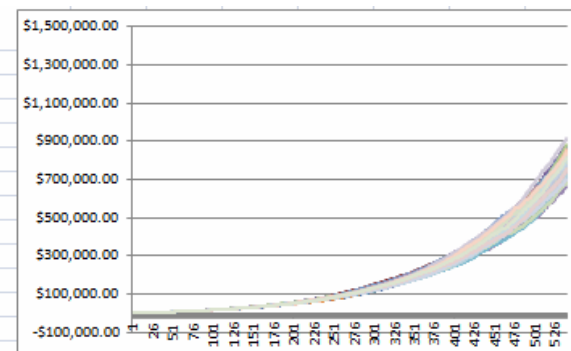
- <http://www.algebra.com/algebra/homework/word/finance/Compound-Interest.lesson>



- [http://en.wikipedia.org/wiki/E_\(mathematical_constant\)](http://en.wikipedia.org/wiki/E_(mathematical_constant))

Monthly Contribution	100
Average Annual Rate of Return (after tax)	8
Annual Contribution increase rate	3

Average of 1000 trials	
\$	780,990.09
Minimum of 1000 trials	
\$	937,514.71
Minimum of 1000 trials	
\$	627,759.78



		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
\$	774,703.31	100	200.82	303.67	405.89	510.97	617.6	723.49	832.05	935.81	1048.13	1162.94	1270.48	1390.23	1510.15	1625.83	1733.21	1847.08	1952.29	2058.92	2179.01	2296.9
\$	786,004.10	100	200.89	303.02	404.21	507.69	609.73	715.07	825.04	931.89	1040.64	1147.07	1252.27	1356.2	1464.82	1577.04	1686.6	1795.91	1910.36	2034.47	2145.41	2255.96
\$	793,677.78	100	201.19	301.83	405.67	511.59	614.49	721.77	828.7	935.29	1046.44	1149.46	1259.98	1373.12	1484.58	1599.89	1706.89	1821.94	1949.39	2064.02	2191.16	2314.12
\$	727,116.87	100	200.45	301.5	405.82	508.91	612.65	714.88	820.24	929.04	1033.45	1137.22	1254.78	1367.18	1476.41	1593.42	1702.66	1810.74	1932.47	2049.28	2174.09	2297.46
\$	752,767.78	100	200.55	302.46	406.71	510.08	614.85	718.06	829.2	935.72	1043.78	1149.36	1259.71	1374.88	1483.96	1588.03	1698.63	1815.64	1921.6	2032.4	2157.41	2270.51
\$	738,548.18	100	200.48	301.72	403.71	507.02	610.34	712.61	817.88	926.98	1033.97	1146.79	1264.09	1379.87	1493.24	1616.37	1737.99	1855.91	1979.37	2091.21	2218.79	2343.95
\$	729,034.32	100	200.67	302.51	403.88	508.15	615.22	722.28	830.28	933.89	1044.54	1150.37	1254.93	1364.95	1473.92	1581.61	1693.11	1802.71	1910.72	2019.85	2146.1	2270.19
\$	849,628.17	100	200.25	301.62	404.31	509.77	615.92	724.84	833.88	945.58	1051.98	1166.26	1280.77	1398.58	1506.12	1624.8	1743.27	1865.68	1988.53	2114.8	2233.22	2366.12
\$	786,475.15	100	200.46	302.65	403.58	507.36	612.35	716.39	826.09	938.52	1050.61	1157.93	1267.33	1377.73	1493.51	1603.76	1712.83	1820.01	1931.5	2045.6	2156.44	2265.85
\$	766,840.72	100	200.95	303.65	405.79	508.55	613.94	717.42	827.52	932.39	1043.49	1149.2	1256.95	1363	1474.07	1584.06	1704.07	1830.79	1940.16	2067.59	2194.11	2318.97
\$	744,404.36	100	201.08	302.13	405.6	508.77	616.25	723.75	831.59	935.65	1039.96	1151.06	1255.31	1361.93	1483.27	1595.32	1701.3	1827.45	1942.15	2062.33	2180.72	2302.11
\$	796,193.45	100	201.16	303.44	408.09	509.27	617.28	724.18	830.9	943.01	1052.59	1164.88	1278.14	1391.26	1498.68	1620.74	1740.32	1852.45	1978.48	2098	2227.25	2348.97

Thanks for all the
work you do with students!

To get a copy of this powerpoint, and for other
Common Core math resources, go to:

<http://ccssmlibrary.blogspot.com/>